A Lewisian Semantics for the English Definite Determiner

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Abstract

We propose a semantics for the English definite article "*the*" which relies on the notion that conversational participants can make rational inferences about which individuals in the conversational context are more contextually salient. Following Frank and Goodman (2012), we assume the contextual salience of an individual is the prior probability that conversational participants will refer to the individual. Our hypothesis is that "*the* NP" refers to the individual which is a member of the set denoted by the NP and has a higher contextual salience than all other members. The property of referencing the contextual salience of the members of its restriction set distinguishes "*the*" from other kinds of English quantificational determiners such as "*every*", which we hypothesise does not have this property. We therefore make different predictions from approaches which state that sentences with definite plurals are truth conditionally equivalent to universal quantification (e.g., Link 1983). We confirm the differing behaviours of "*the*" and "*every*" experimentally.

Keywords: definites; plurals; quantification; salience

In this paper we argue for a particular view of the semantics of definite determiners which relies crucially on a refined notion of contextual salience as defined by Frank and Goodman (2012). Their approach gives a rigid definition to the notion of contextual salience of an object as the prior probability that conversational participants will refer to that object. We have derived a theory of definite descriptions which incorporates this definition of contextual salience. Under our theory, a definite determiner is a function which takes a set of objects denoted by the determiner's NP complement, and returns the most contextually salient member of that set.

The semantics we propose shares much with the theory of definites proposed by Lewis (1979), who argues that the semantics of definites must refer to a contextually determined ranking of individuals by their salience. This study addresses some central and interesting broader questions in natural language semantics. To what extent do particular lexical items refer to the contextual salience of individuals? Can a refined notion of contextual salience shed light on the semantics of definiteness in natural language?

Frank and Goodman (2012) address central issues in natural language pragmatics by proposing a quantitative model of the rational inferences a language interpreter make about reference. Their assumption is that interpreters use Bayesian inference in determining the intended referent of an utterance.

Their model gives $P(r_s|w,C)$, the probability that a speaker's intended referent is r_s given a word w in a context C. The expression is the product of the contextual salience of r_s and the informativity of w in C, normalised by the sum of these terms for each referent in C.

Explicit in the formulation is that speakers are able to make rational inference about the contextual salience of objects in C, and that this property of an object may be quantified. This idea imposes a worldview in which some objects are more contextually salient than others, and that in at least some contexts, there is a most contextually salient object within a set of objects.

In their model, the contextual salience of an object is assumed to be the prior probability that conversational participants will refer to that object. The definition is purposefully vague between whether something is salient perceptually or obtains salience over the course of the conversation. It is also insensitive to whether the reference will be made by any particular conversational participant (speaker or listener).

The following section relays some prominent theories of the semantics of the English definite determiner "*the*". Most pertinently, the proposal by Lewis (1979), who proposes that the semantics of "*the NP*" should presuppose a way of ranking the contextual salience of the objects denoted by the NP. We suggest the Frank and Goodman (2012) methodology for determining contextual salience and then explain how we have and will continue to confirm these predictions experimentally.

Definiteness

Within any discussion of the semantics of definites, there is a discussion of whether a definite description, like a "*the NP*" phrase, refers to an individual or a generalised quantifier. The approach in Montague (1973) is to characterise all NPs as generalised quantifiers, including definite descriptions. Montague defines the semantics of "*the*" via a syncategorematic rule which results in the following definition for a "*the NP*" phrase.

(1) $[\![the\ NP]\!]$
$= \lambda P.\exists y[\forall x[[\![NP]\!](x) \leftrightarrow x = y] \wedge P(y)]$
= the set of properties P such that there is a sole member y in $[\![NP]\!]$, and P is true of y.

The rule may operate on any $[\![NP]\!]$ and is therefore a total function. It denotes the set of properties held by the one and only member of the set of $[\![NP]\!]$. Therefore, $[\![the\ circle]\!]$ denotes the set of properties of the one and only contextually relevant circle. If a sentence has the $[\![\textit{the circle}]\!]$ taking widest scope, it will always be false if there is no unique circle.

Partee (1987) proposes that total functions on $[\![NP]\!]$ properties which return definite descriptions, like Montague's definition of "*the*" in (1), co-occur with partial functions. Partee describes the ι function, the partial surjective function which maps any singleton set to its sole member. Any application of $\iota(P)$ where P is non-singleton will be undefined. A semantics for "*the*" based on the ι function:

(2) $[\![\textit{the NP}]\!]$
$= \iota x.[\![NP]\!](x)$
$=$ the unique x such that $[\![NP]\!](x)$ is true

The total function in (1) which Partee labels THE and the function ι in (2), are both claimed by Partee to be equally viable definitions of "*the*". Partee defines a system where lifting and lowering operations between an individual type and a generalised quantifier render the definitions of $[\![\textit{the NP}]\!]$ in (1) and (2) alternations of one another that an interpreter is equally able to access in the computing of a sentence.

We take no particular stand on whether definite descriptions have flexible types. However for the purposes of our discussion, we assume that definite descriptions denote individuals. If they ever denote generalised quantifiers, we assume these interpretations are derived from the basic type via a lifting operation.

Explicit in both kinds of semantics ascribed to $[\![\textit{the NP}]\!]$, $\iota([\![NP]\!])$ or $THE([\![NP]\!])$, is that only where $[\![NP]\!]$ denotes a singleton set can a sentence containing the expression be true. For many scenarios, this characterisation seems intuitive. The following sentence seems ill-formed precisely because "*the table*" has an unclear referent.

(3) #Jane entered the café and looked around. The table was slightly wobbly.

In (3) we imagine an ordinary context in which the café contains a non-singelton set of tables. The characterisation of "*the table*" as $\iota([\![table]\!])$, captures the oddness. As ι is a partial function with only singleton sets in its domain, $\iota([\![table]\!])$ is undefined as $[\![table]\!]$ is a non-singleton set. Contrast (3) with (4), which intuitively sounds more well-formed.

(4) Jane entered the café and looked around. She sat down. The table was slightly wobbly.

Why does the additional sentence in (4) alleviate the oddness of (3)? One approach is to deny the other tables are contextually relevant. Lewis (1979) gives the following example addressing this concern. He asks the reader to imagine a context in which a particular cat, Bruce, is perceptually salient (perhaps pacing in front of us).

(5) The cat is in the carton. The cat will never meet our other cat, because our other cat lives in New Zealand. Our New Zealand cat lives with the Cresswells. And there he'll stay, because Miriam would be sad if the cat went away.

Here we have a discourse with (at least) two contextually relevant cats, yet within the discourse, the usage of "*the cat*" is not only felicitous but also picks out two different cats at different points.

Based on examples like (5), Lewis proposes that "*the NP*" denotes some object x if and only if x is the most salient member of *NP* in the discourse. He therefore presupposes that conversational participants must be able to infer a contextually determined ranking of the members of the set $[\![NP]\!]$ in order to determine the *most* salient member.

Lewis' model accounts for the (3) and (4) contrast nicely. Without the information conveyed by the sentence "*She sat down*", the contextually relevant tables are *prima facie* equally ranked for contextual salience. The Lewisian semantics for "*the*" predicts a presupposition failure in this scenario. The addition of "*She sat down*" raises the contextual salience of Jane's chosen table such that it outranks all other tables in the café. The presupposition that $[\![table]\!]$ contains a most salient member is satisfied.

Lewis' outstanding issue in his analysis is his notion of contextual salience, which he leaves undefined. We propose that the Frank and Goodman (2012) definition of contextual salience serves well here. By assuming that speakers are able to infer the probability of reference of each object in a context immediately gives Lewis his ranking of contextual salience. Each member of a set $[\![NP]\!]$ is assigned a probability and thereby a ranking is imposed.

Our semantics for "*the*" is a function from a set of individuals to the most salient individual member of that set. The analysis shares with Partee that $[\![the]\!]$ is a partial function, which is only defined where its argument contains a unique most salient member. Where it departs from Partee is the requirement that its argument must be a singleton set.

(6) $[\![\textit{the NP}]\!]$
$= \iota x[[\![NP]\!](x) \wedge \forall y[[\![NP]\!](y) \rightarrow P(r_x) \geq P(r_y)]]$
$=$ the unique x such that $[\![NP]\!](x)$ is true and x is the most salient member of $[\![NP]\!]$

In the next section, we extend this analysis to give a semantics for definite plurals and argue that they are not truth-conditionally equivalent to universal quantification.

Plurals

Both Partee (1987) and Lewis (1979) are vague about how their theories of definite singulars translates to a theory of definite plurals. In this subsection we lay out the dominant theory of the semantics of plurals stemming from Link (1983) and discuss how it informs a theory of definite plurals.

Link defines an operation on a set of individuals $[\![P]\!]$ labelled as $[\![^{*}P]\!]$. We will relabel it $[\![\textsc{sums}(P)]\!]$ for perspicuity. $[\![\textsc{sums}(P)]\!]$ is the set of all individual sums of members of $[\![P]\!]$, thus forming a complete join-subsemilattice of $[\![P]\!]$. If the extension of $[\![P]\!]$ is as in (7), then the extension of $[\![\textsc{sums}(P)]\!]$ is in (8)

(7) $[\![P]\!] = \{a,\ b,\ c\}$

(8) $[\![\text{SUMS}(P)]\!] = \{a,\ b,\ c,\ a \oplus b,\ a \oplus c,\ b \oplus c,\ a \oplus b \oplus c\}$

Link also defines the operation on $[\![P]\!]$ labelled as $[\![^{\odot}P]\!]$, which we will label $[\![\text{PL}(P)]\!]$. $[\![\text{PL}(P)]\!]$ is the set of non-atomic sums in $[\![\text{SUMS}(P)]\!]$.

(9) $[\![\text{PL}(P)]\!] = \{a \oplus b,\ a \oplus c,\ b \oplus c,\ a \oplus b \oplus c\}$

The final relevant definition given in Link (1983) is the *proper sum*. The *proper sum* of $[\![P]\!]$ is the supremum of all objects in $[\![\text{PL}(P)]\!]$. In our model, the *proper sum* of $[\![P]\!]$ is $a \oplus b \oplus c$. The *proper sum* is a partial function, only defined for non-singleton $[\![P]\!]$. Link associates the *proper sum* of $[\![P]\!]$ with the definite plural:

(10) $[\![\textit{the Ps}]\!]$
= $\iota x[x \in \text{PL}(P) \wedge \forall y[y \in \text{SUMS}(P) \rightarrow y \oplus x = x]]$
= the *supremum* member of $[\![\text{PL}(P)]\!]$

Disregarding issues of distributivity, the semantics Link provides for definite plurals means that a sentence with a widest scope definite plural is truth-conditionally equivalent to a universally quantified sentence.

Our semantics gets a different result. We assume that $[\![\text{PL}(P)]\!]$ in (9) is the right semantics for the plural version of P. Our semantics for "*the*" in (6) pics out the most salient non-atomic sum in $[\![\text{PL}(P)]\!]$. This analysis presupposes that speakers may make inferences about the relative contextual salience of sums of individuals as well as atomic individuals. Consider the following discourse.

(11) In the café, an angry toddler threw around his spaghetti near where he was sitting. After he left, the waitress came and wiped the tables.

In this example "*the tables*" may denote less than all the contextually relevant tables in the café. We believe this is predicted by our semantics in which a non-supremum sum of café tables (those dirtied by spaghetti) outranks the supremum (the sum of all tables in the café) in terms of their contextual salience.

Given this kind of analysis, we predict that definite plurals are truth conditionally distinct from universal quantification. It is this claim that the experimental component of this paper has targeted.

Experiment

We are conducting several experiments testing the validity of our hypotheses. The first experiment we conducted is reported in this paper. In this experiment, we tested whether speakers consider the relative contextual salience of members of a set of individuals when calculating the truth values of sentences with definite plurals and universal quantification.

Our initial results provide support for two claims. Firstly, in identical contexts, speakers were unwilling to judge sentences with "*every*" as true if there were any counterexamples, however they were willing to judge sentences with definite plurals as true with large numbers of counterexamples. We argue that our proposed semantics for the definite determiner predicts this outcome.

Secondly, speakers seem to ignore the relative contextual salience of members within the restriction set of "*every*", but do consider relative contextual salience of members within the restriction set of "*the*".

We tested these hypotheses with variants on stimuli such as in Figure 1:

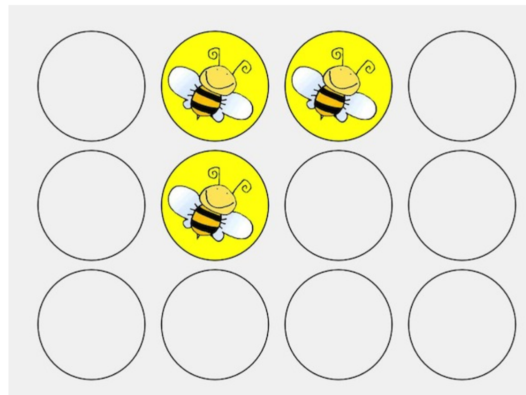

Figure 1: Sample scenario

Participants were presented with 11 variations on Figure 1, all of which contained 3x4 arrangements of equal sized circles. We varied whether all twelve, none of the twelve or exactly three circles were coloured yellow. We varied whether all twelve, none of the twelve or exactly three circles contained a drawing of a bee inside them. In cases where there were exactly three yellow circles and exactly three circles with bees, we varied whether these were the same three circles, non-overlapping sets of three circles, or sets which overlapped by exactly two.

Our intention was to configure yellowness as a property which would make circles more salient (yellow circles being more salient than blank circles). In all stimuli, speakers were asked to judge whether or not circles contained a bee.

Each participant was presented with all 11 possible scenarios. They were presented with a sentence for each scenario which they were asked to rate from 1 to 6 for truth or falsity (1 being *definitely false* to 6 being *definitely true*). The sentence for each scenario was chosen at random for each participant between three options:

(12) *Every circle is occupied.*

(13) *The circles are occupied.*

(14) *No circle is occupied.*

We explained to the participants in the instructions that whenever a circle had a bee inside it, we referred to the circle as being "*occupied*". We decided on the particular wording of "*is/are occupied*" as we wanted to carefully avoid using the expression "*a bee*" in the stimuli to avoid any interference from quantifier scope ambiguity effects. We also wanted to avoid any stimuli which contained anaphoric pronouns such as "*every circle has a bee in it*", as the pronouns would become plural under the condition using "*the circles have a bee in them*", thus becoming a less satisfying minimal pair.

In subsequent versions of this experiment, we will phrase the stimuli differently. Audience responses post-survey claimed that at some points they considered yellow circles with no bee to be "*occupied*" (by yellowness), despite our explanation in the instructions.

We also considered our choice of properties problematic. Under the current design, both yellowness and bee-containment are properties which could ostensibly influence whether speakers consider a circle "salient". An experimental design in which the property that participants are asked to judge as being true or false does not also induce a saliency effect will have clearer and more-interpretable results.

Once we have established the best possible design of this experiment, we intend to independently measure the salience of the set of twelve circles by replicating the methodology in the Salience Condition in Frank and Goodman (2012). We will give participants the same set of stimuli from our first experiment and ask participants to bet an amount of money on each circle based on the probability that someone referred to that circle. The instructions would ask the participant to imagine that someone referred to one of the circles using an unfamiliar word and to bet on which circle was being talked about.

By the reasoning in Frank and Goodman (2012), for any object x, this task gives the prior probability $P(r_x)$ that x will be referred to. This gives us an empirical way of measuring the contextual salience of an individual object. We hypothesise in a scenario like Figure 1, that the yellow circles containing bees will receive higher bets than the blank circles. This will give a numerical value for each circle which we refer to as its *saliency value*. Our Lewisian semantics for a singular definite "*the circle*" will be the unique circle which has the highest saliency value. For any scenario where circles receive equal saliency values will lead to presupposition failure — there will be no unique circle with a highest saliency value.

Recall our assumed semantics for plurals that we laid out in the previous section. We consider an NP with plural morphology to denote the set of non-singleton subsets which demonstrate the property denoted by the NP. The NP "*circles*" denotes all possible non-singleton subsets of a contextually relevant set of circles. Combining this insight with our assumed methodology for measuring salience is the crucial complexity of this next experiment: how do we define a task which asks participants to reliably judge the most salient non-singleton subset of objects?

One idea is to diverge from the betting methodology in Frank and Goodman (2012). We present experimental participants with the eleven stimuli. They would again be told to imagine someone referred to a set of circles and to click on at least two (but possible more) circles which they believe the person was referring to. We do not record how often each circle is chosen, rather, we record how often each combination of circles is chosen. This gives us a *saliency value* for each subset of circles, and allows the semantics we have defined for "*the circles*" to be the subset of circles with the highest saliency value.

This methodology forces each participant to make a categorical choice about salience, where the Frank and Goodman (2012) allows participants to make graded judgements. This is a practical choice as the task of assigning a bet to each non-singleton member of the powerset of circles quickly becomes unfeasible with bigger sized stimuli.

Participants: We gave the experiment to 109 speakers of English online. We did not ask about any age or gender information.

Hypothesis 1: the semantics of definite plurals is not equivalent to universal quantification. We assume the following semantics for universal quantification and definite plurals respectively.

(15) $[\![\text{every circle is occupied}]\!]$
$= \forall x[\text{circ}(x) \rightarrow \text{occ}(x)]$

(16) $[\![\text{the circles are occupied}]\!]$
= the most salient $x \in \text{PL}(\text{circle})$

In the latter definition we depart from previous works such as Link (1983), who gives the following definition.

(17) $[\![\text{the circles are occupied}]\!]$ (Link 1983)
= the supremum member of PL(circle)

We predict that participants will give unequal judgements to sentences containing widest scope definite plurals (*The circles are occupied*) vs. widest scope universal quantification (*Every circle is occupied*).

Given (15), we predict that participants will only judge a universally quantified sentence as true if all contextually relevant circles are occupied. Within the no saliency condition (no circles yellow) and all circles occupied, participants were categorical in their judgements, judging the sentence *every circle is occupied* with a mean score of 6 out of 6 (*definitely true*).

In scenarios with no saliency (no yellows), and exactly three circles occupied, we expect participants to judge *every circle is occupied* as false (given the presence of counter examples). Indeed, they were almost categorical with a mean score of 1.08 (*definitely false*).

Given the same scenarios, a supremum analysis of definite plurals (17) predicts similar results for the same scenarios paired with the sentence *the circles are occupied*. Participants were almost categorical when all circles were occupied (5.95). However they hedged far more when exactly three circles were occupied (3.67). The variance between the no-yellow, some-occupied condition for *the* and *every* were highly significant (p = 5.14E-12), using the Student's Two-Tailed T-test to measure significance.

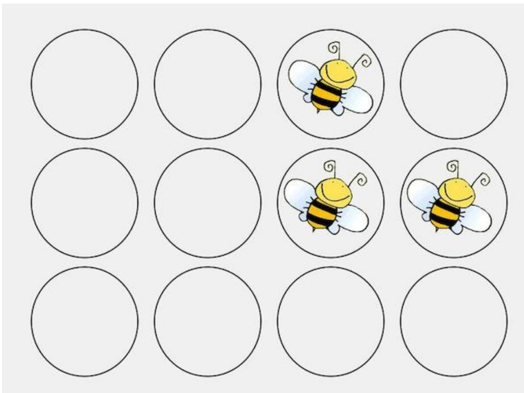

Figure 2: No-yellow, some-occupied

We analyse this result as participants judging the sum of the three occupied circles as more contextually salient than the supremum sum of the whole circle set (judging $P(r_{b1\oplus b2\oplus b3}) > P(r_{supremum})$ as true, where b_{1-3} refers to the occupied circles). The participants therefore judge "*the circles are occupied* as true, considering the formula in (16).

Hypothesis 2: speakers ignore the relative contextual salience of the circles when judging sentences with *every*, but do consider relative contextual salience of the circles when judging definite plurals.

In considering this hypothesis we looked at scenarios with no salience (no yellows) and some occupied circles as in Figure 2. We compared these with the some-salience (three yellows) and some-occupied condition, in which the three yellow circles and the three occupied circles were non-overlapping sets, as in Figure 3.

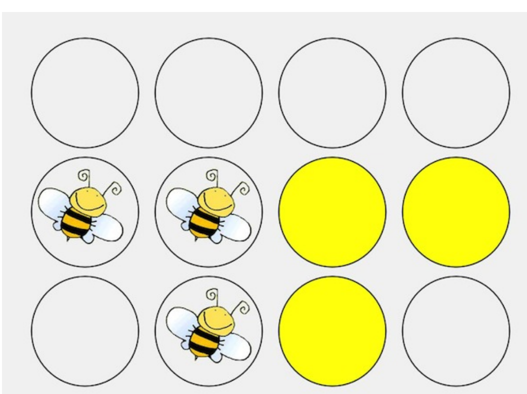

Figure 3: Some-yellow, some-occupied, exclusive

Given the semantics of *every* in (15), both scenarios in Figure 2 and Figure 3 should be judged as categorically false. This is correct, with respective means of 1.08 and 1.06. The variance between them was not significant (p = 0.738).

The supremum analysis of definite plurals would predict that participants judge both scenarios in Figure 2 and Figure 3 as false when presented with a definite plural, with no variance between them. We instead find significant variance, with respective means of 3.67 and 2.98 (p = 0.0396).

As above, we analyse the participants as judging the three occupied circles in Figure 2 as more contextually salient than the supremum sum of the whole circle set. In Figure 3 however, the three occupied circles are competing with the three yellow circles. We analyse the drop in truth judgements as participants positing that the yellow circles are more salient than or equally salient to the occupied circles (judging $P(r_{y1\oplus y2\oplus y3}) \geq P(r_{b1\oplus b2\oplus b3})$ as true). The scenario therefore does not satisfy (16) via presupposition failure. There is no most salient set.

Conclusion

We proposed a semantics for the English definite article which captures the central insight of Lewis (1979), that the article imposes a ranking of individuals in the restriction set by their contextual salience and the definite description denotes the most salient individual. We extended this hypothesis by (a) proposing a way that it could extend to definite plurals, by ranking the non-atomic sums of individuals and (b) giving a rigid definition of contextual salience taken from Frank and Goodman (2012). We conducted an experiment demonstrating some advantageous results of our theory, specifically concentrating on the prediction that our semantics for definite plurals is not equivalent to universal quantification. We confirm that participants give highly significant differences in judgement between the two types of quantification, and further that participants are sensitive to the contextual salience of individuals when judging definite plurals.

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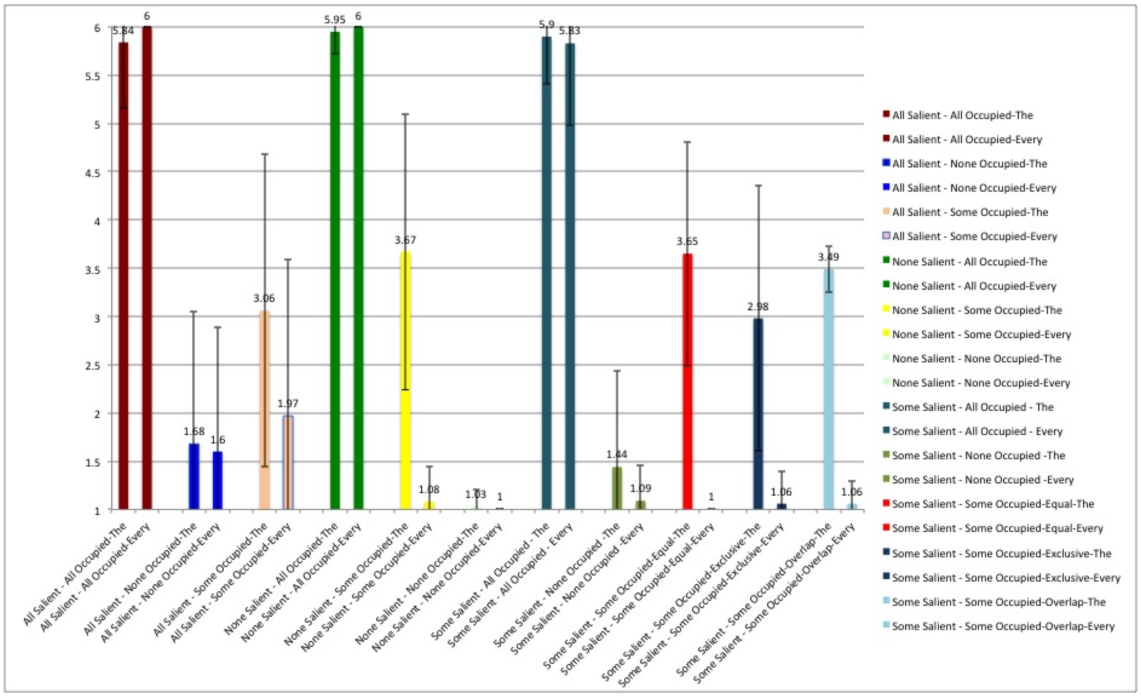
Appendix



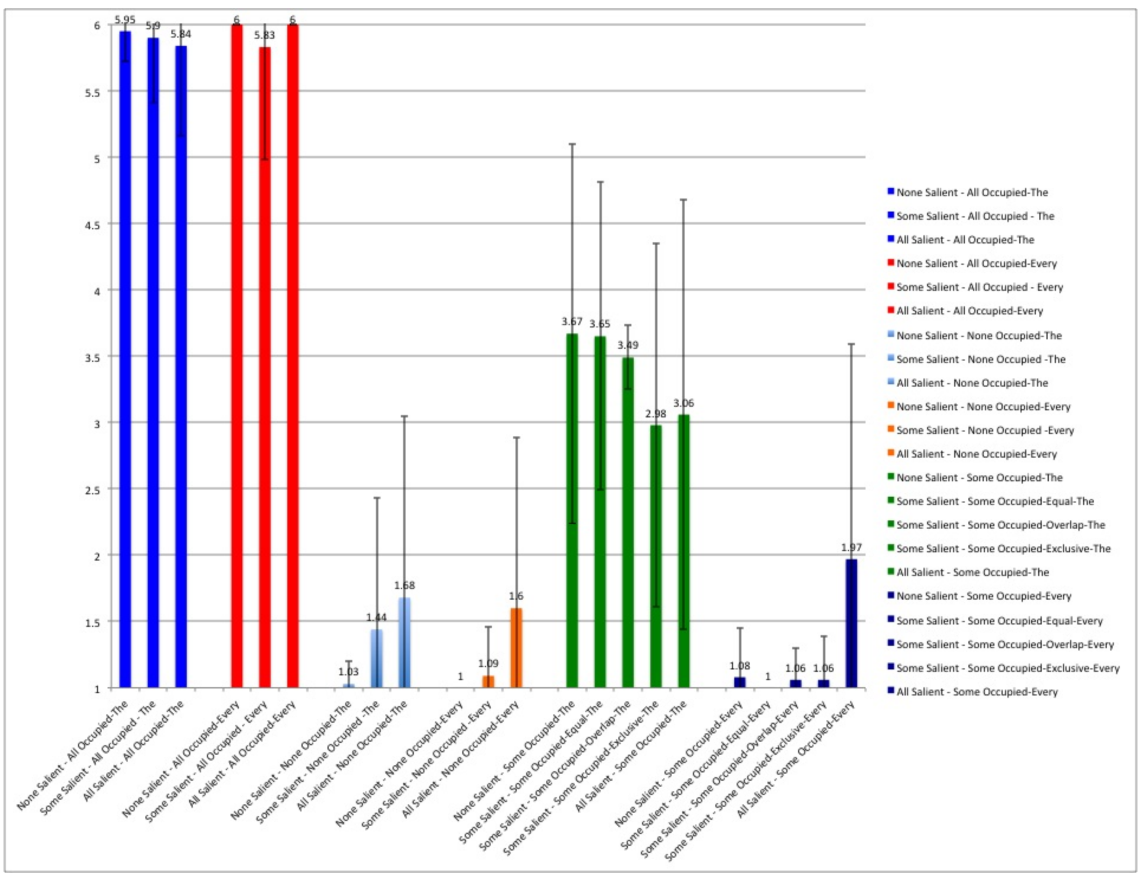


Figure 4: *the* vs. *every* for each condition (top)
Variance in salience for each condition (bottom)